Serafina Cuomo, *Pappus of Alexandria and the Mathematics of Late Antiquity*. Cambridge (Cambridge Classical Studies, Cambridge University Press) 2000. pp. ix + 234, including 22 pp. bibliography, 4 pp. general index and 7 pp. index locorum.

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Compared to the many pages devoted to the mathematical heroes of the early Hellenistic age, relatively few have dealt with the mathematical authors of the Roman period, from Hero to Eutocius (astronomy excluded). Similarly, the actual mathematical theorems, constructions and proofs of ancient mathematicians has been dealt with much more extensively and profoundly than the characteristics of the professional environment, the norms governing their work and the general epistemological orientations of ancient mathematicians of all epochs.

In recent years, several large-scale works have appeared where the mathematics of later Antiquity are in focus or play a larger role. With few exceptions, however, they have concentrated on mathematical contents; by attempting a global characterization of late ancient mathematics as reflected in Pappus's *Mathematical Collection* and a large number of other sources, Serafina Cuomo's volume thus provides important and innovative insights.

Chapter 1 ("The outside world") describes the general social setting of mathematics in later Antiquity and the whole range of professions which at the times were seen (or saw themselves) as linked to mathematics: astrologers; land-surveyors, architects, mechanicians and similar technicians; accountants at different levels; teachers of these, of philosophy as part of general upper-class culture, and of theoretical mathematics proper. Drawing extensively both on writings made by members of these groups and on legal and other external sources, it is shown, firstly, that mathematical activities were quite visible in the cultural landscape, and, secondly, that these activities, like other intellectual activities, were organized as clusters "delimited by boundaries, although these boundaries may be blurred and in need of being constantly redrawn" (p. 55). Divisions, it is further argued, were not "determined exclusively by the type or extent of knowledge [involved but also on the use made of such knowledge], on the *ethos* that accompanied the profession [...] and on its *utilitas* (interpreted in a wide sense as advantage, common benefit)".

Chapters 2 through 4 analyze the organization and argumentative style of selected books of the *Collection* closely and conclude convincingly that they address different audiences and do so with specific agendas. In chapter 2, book 5 (on isoperimetric problems and regular polyhedra) is seen to address an "implied reader" who (in strong contrast to the implied reader of other books) is generally educated but more familiar with the Neoplatonic and Neopythagorean than with the mathematical tradition, and to be intended as an argument in a "culture war", be it specifically against "Iamblichus or his followers", be it against a "straw adversary, who embodied trends current at the time" (p. 84) – Pappus's point being that the philosophers flaunt their knowledge of the obvious as "earth-shattering revelations" (p. 86) and fail when it comes to prove the substantial. At the same time, as argued by Cuomo, the "deliberately oversimplified organization" of the material tends to make the lay reader see Pappus not only as a competent master but "as a master whose knowledge cannot be shared to its full extent" (p. 87).

Chapter 3 looks at book 8, dealing with mechanics – treated as the "Cindarella of Greek science" by a historiography whose appreciation of the ancient attitude to applied knowledge is based on Plato, Xenophon, Aristotle, Plutarch and their kin. Without denying the significance of the "mainstream view" – the more or less outspoken disdain for material practice and 'productive knowledge' – Cuomo argues from a wide range of sources (including ps-Aristotle's *Mechanical Questions* and genuine mechanical writers as Philo of Byzantium and Hero) and specialized studies that differences depending on location and epoch should not be left out of view, and that the contrast between theoretical and mechanical knowledge "should not be cut in clear-cut terms, as upper class versus lower classes, high culture versus popular culture or written tradition versus orally transmitted forms of knowledge"; mechanics, indeed, "was associated *both* with τέχνη and the crafts *and* with power and the mightiest human representatives of power" (p. 95).

Pappus himself, it is shown, presents mechanics as a theory with supreme prestige, but also as ἐπιστήμη (in which respect it includes geometry) and τέχνη united, ideally to be mastered by the same person (which, however, the immensity of the knowledge required will mostly prevent – Pappus himself claims no practical experience). The praise of the subject is actually so emphatic that it "makes one wonder whether he is not overdoing it in order to counterbalance possible detractors" (p. 108) –

to which one might add that the Neoplatonic and Neopythagorean adversaries from book 5 will certainly have held the mainstream view.[1]

Whereas many early writings on mechanics were addressed to politically potent patrons and correspondingly oriented toward military technology and wonder-working, Pappus's book 8 is seen to be addressed to private citizens and to possess a broader view of the *utilitas* of the topic. His

> defence of the indissolubility of mechanics and mathematics is not only a tribute to authoritative traditions, but also a claim to a wider source of legitimation for mathematicians. He stresses the role of geometry as *both* educationally and culturally useful (embodied by the scholar as mouthpiece of the tradition) *and* materially useful (personified in the able architect / engineer / mathematician) (p. 126).

Chapter 4 moves between books 3, 4 and 8, focusing on the distinctions between "mechanical" and geometrical methods and between planar, solid and linear problems and methods, as applied to the problems of two mean proportionals and the division of the angle, and the treatment of the "linear" curves spiral, cochloid and quadratrix. Cuomo analyzes Pappus's explicit judgment of the ways of predecessors and contemporaries and the implicit attitudes expressed in his reporting of some of the known solutions and omission of others, extracting thus Pappus's own norms for how problems should be solved and his notion of legitimacy. The approach to the tradition is seen to be "not so much cut-and-pasted as tailor-made; it only comes to life when it takes on a certain guise to serve a particular purpose" (p. 168); in

> the case of the two mean proportionals, he is keen to stress his role as custodian and successor of the earlier geometers, so that his direct interventions are very explicit; he criticizes in detail what he takes to be a misguided attempt and presents his own contribution to the topic in such a way as to make it the culmination and compendium of previous efforts. In the case of linear curves, instead, his subject needs consolidation, so Pappus's main focus is to present the curves as effective problem-solving tools, whose utility is proved by applying them to a number of constructions, and whose homogeneity is underscored by

[1] Even though, paradoxically, the mathematical competence of the Neopythagorean writers will mostly have been insufficient to allow them to appreciate the Euclidean and higher mathematical levels, for which reason the *Theologumena arithmeticae* and similar writings are often our best sources for the knowledge that circulated among mathematical practitioners (without telling the sources for their stupendous insights).

streamlining their definitions and the description of their main properties.

Chapter 5 summarizes what can be said about Pappus's general agenda (without deciding whether it is a personal agenda or borrowed from some lost source). His interest both in generalization and in the systematic analysis of sub-cases is shown not to be cheap tricks that allow a second-rank mind to feel original but to express a particular metatheoretical awareness, for instance in "how a change in some of the elements, or a change in the relation between the geometrical objects in question, affects the demonstration or construction" (p. 176); the approbation of technical mathematics is seen to be accompanied by a preference for practically feasible mechanical methods over impractical though theoretically more satisfactory solid constructions and by willingness to use numerical arguments (which are more than mere examples). The explicit norm that (e.g.) solid problems should be solved by solid, not linear constructions is seen to reflect the emphasis on appurtenance to the professional tradition – namely because (until the nineteenth century) the only available criterion for this classification was how the tradition had henceforth been able to treat the problem in question. But appurtenance to the tradition entailed no compiler's blind reverence – Pappus might well criticize Archimedes for metatheoretical misbehaviour and Apollonius for undue arrogance toward the predecessors on whose shoulders he stood. In a way, his view of the past was "whiggish", and like more recent whiggism thus an expression of a present time very conscious of its own qualities.

Similar relating oneself to the tradition through "[c]ommentaries, introductions, biographies" was common practice in late ancient "fields such as philosophy, grammar and medicine; law became more and more officially constituted as a practice and a form of knowledge where tradition played a primary role" (p. 200). As summarized in the final lines (p. 201), "Pappus is a part of [the cultural life of the fourth century] and needs to be seen as such. His cultural context helps to explain Pappus, and he contributes to that context himself in showing how mathematics was part of the picture".

The book is well written in pleasant style. The argument is based throughout on a wide range of sources, and the pertinence of the source and the leap from source to interpretation is always made clear. The reviewer felt well treated and entertained while reading and quite a bit wiser when closing the book.

Jens Høyrup